CEPI PAKISTAN COMPETITIVE EXAMS PREPARATORY INSTITUTE

FEDERAL PUBLIC SERVICE COMMISSION

Roll Number

COMPETITIVE EXAMINATION-2026 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: **(i)** Attempt only **FIVE** questions in all**. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

Q. No. 1 **(a)** Prove $\nabla f(r) = \frac{f'(r)}{r}\vec{r}$, and evaluate U for $\nabla U = 2r^4\vec{r}$. **(8)**

(b) If $\vec{A}$ and $\vec{B}$ are differentiable vector function of position (x, y, z), find $\nabla . (\vec{A} \times \vec{B})$. **(7)**

(c) If $\vec{A} = x^2yzi - 2xz^3j + xz^2k$ and $\vec{B} = 2zi + yj - x\text{^}2k$, find $\frac{\partial^2}{\partial x\, \partial y}(\vec{A} \times \vec{B})$. **(5)**

Q. No. 2 **(a)** What is the maximum range possible for a projectile fired from a cannon having muzzle velocity 1 mile/sec? What is the height reached in this case? **(10)**

(b) Find the radial and transverse components of the velocity of the particle moving along the curve $ax^2 + by^2 = 1$ at any time t, if the polar angle $\theta = ct^2$. **(10)**

Q. No. 3 **(a)** Verify that the linear differential equation **(5)**

$$(1 + x^2)\frac{dy}{dx} + 2xy = e^{x^2}$$

can be solved by setting $y = ve^{x^2}$. Find the equation satisfied by v.

(b) Explain how a first-order linear differential equation can be used to model pollution accumulation in a lake with constant inflow and outflow rates. **(5)**

(c) Find the solution of the following differential equation **(10)**

$y''' + 8y = 2x - 5 + 8e^{-2x}$; with the conditions

$y(0) = -5, \quad y'(0) = 3, \quad y''(0) = -4$

Q. No. 4 **(a)** Find the general solution of the following differential equation on interval $(0, \infty)$ **(10)**

$$16x^2y'' + 16xy' + (16x^2 - 1)y = 0$$

(b) Solve the following PDEs using transformation **(10)**

$$u_t + u_x + u = 0\,; \quad for\ x > 0, t > 0,$$

$$u(0,t) = sin(t); u(x,0) = 0.$$

Q. No. 5 **(a)** Model the one-dimensional wave equation. **(8)**

(b) A stretched string of length L is lying along x-axis and is fixed at both ends $x = 0$ and $x = L$. Find the deflection $u(x,t)$ of the string at any time t, if initial displacement $u(x,0) = f(x) = Lx - x^2$ and the initial velocity is **(12)**

$u_t(x,0) = g(x) = 4.$

Q. No. 6 **(a)** Find the Fourier series of the function **(10)**

$$f(x) = \left(1 - \frac{|x|}{a}\right) H\left(1 - \frac{|x|}{a}\right),$$

Where $H(x)$ is the Heaviside unit step function defined by

$$H(x) = \begin{Bmatrix} 1, & for\ x > 0 \\ 0, & for\ x < 0 \end{Bmatrix}$$

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(b) Evaluate the integral $I = \int_0^3 \frac{1}{1+x^4} dx$ by using **(10)**

(i) Trapezoidal rule

(ii) Simpson's 1/3 rule by taking h = ¼.

Q. No. 7 **(a)** Use the Modified Euler method to solve the IVP: **(10)**

$$y' = y - x^2 + 1; \quad y(0) = 0.5$$

for $0 \leq x \leq 2$ with step size $h = 0.5$.

Compare with the exact solution.

(b) Implement appropriate technique to solve the following linear system. **(10)**

$$3x_1 - x_2 + x - 3 = -1$$
$$-x_1 + 3x_2 - x_3 = 7$$
$$x_1 + x_2 - 3x_3 = -7$$

Q. No. 8 **(a)** Set up Newton's scheme of iteration for finding the square root of a positive number N and evaluate $\sqrt{14}$. **(10)**

(b) Find the first four iterations of the equation $f(x) = x - 0.8 - 0.2sinx$ in the interval $\left[0, \frac{\pi}{2}\right]$ using Newton-Raphson Method. **(10)**

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